1 (e) Linear model $\Rightarrow$ just scale the response by 0.1 (3)

$$\Rightarrow \boxed{V'(t) = \frac{1}{10\rho V_0 C_D A}\left[1 - e^{-\frac{\rho V_0 C_D A t}{m}}\right] \text{ m/s}}$$

The response is plotted in Figure 2

Now $|V'| \ll V_0$, so we expect this result to be physically reasonable.

Problem 2 Solution

2 (a) We have from 1(c)

$$V_s(t) = \frac{1}{\beta}\left(1 - e^{-\frac{\beta}{m}t}\right) \qquad \text{where } \beta = \rho V_0 C_D A$$

and $u(t) = \begin{cases} 0 , & t < 0 \\ 1 - e^{-t} , & t \geq 0 \end{cases}$ so $u(0) = 0$

$$\frac{du}{dt} = \begin{cases} 0 , & t < 0 \\ e^{-t} , & t > 0 \end{cases}$$

Duhamel's superposition integral:

$$y(t) = \int_0^t \frac{1}{\beta}\left(1 - e^{-\frac{\beta}{m}(t-\tau)}\right) e^{-\tau}\, d\tau$$

$$= \frac{1}{\beta}\int_0^t \left(e^{-\tau} - e^{-\frac{\beta}{m}t + \left(\frac{\beta}{m}-1\right)\tau}\right) d\tau$$

$$= \frac{1}{\beta}\left[-e^{-\tau} - \frac{1}{\left(\frac{\beta}{m}-1\right)} e^{-\frac{\beta}{m}t + \left(\frac{\beta}{m}-1\right)\tau}\right]_0^t \qquad \frac{1}{\frac{\beta}{m}-1} = \frac{m}{\beta - m}$$

$$= \frac{1}{\beta}\left[-e^{-t} + 1 - \frac{1}{\left(\frac{\beta}{m}-1\right)} e^{-t} + \frac{1}{\left(\frac{\beta}{m}-1\right)} e^{-\frac{\beta}{m}t}\right]$$

$$= \frac{1}{\beta}\left[1 - e^{-t} - \left(\frac{m}{\beta - m}\right) e^{-t} + \left(\frac{m}{\beta - m}\right) e^{-\frac{\beta}{m}t}\right]$$

From (1): $m = 0.12\text{ kg}$, $\beta = 0.0283 \frac{\text{kg}}{\text{s}}$, $\frac{m}{\beta - m} = -1.31$, $\frac{\beta}{m} = 0.24 \frac{1}{\text{s}}$

$$\boxed{y(t) = 35.3\left(1 + 0.31 e^{-t} - 1.31 e^{-0.24t}\right) \text{ m/s}}$$